

# College Readiness for Mathematics

## Level One: Preparation for General Education Math



**M**athematics instructors from the seven campuses of the University of Maine System have been working together to reach consensus about the skills and competencies that entering students need to have mastered in high school in order to place into and be successful in college level general education math courses in the University of Maine System. General education math courses, while designed for students who do not plan to major in technical or scientific fields, often serve as a gateway to the more advanced math courses that are required in many professional undergraduate programs.

Below we list the mathematical topics and understandings that constitute college readiness for general education mathematics through College Algebra. Topics in italics that are marked with an asterisk are not required for a general education math course, but are required for entry into College Algebra. This document does not address readiness for Calculus I, or other mathematics courses specific to majors in science, engineering and some professional fields.

We recommend deep and broad understandings, rather than mere exposure. Further, we want to stress the importance of both procedural and conceptual understandings. Students who are ready for college mathematics are able to perform mathematical operations and manipulations by hand or with a calculator when appropriate; they understand basic concepts and definitions; and they are able to apply, interpret and communicate results.

The topics and understandings for mathematic readiness are organized under five major headings: Mathematical Reasoning, Computation, Algebra, Geometry, Data Analysis and Statistics.

## **I. Mathematical Reasoning**

**A. Successful students know important definitions, why definitions are necessary and are able to use mathematical reasoning to solve problems; they**

- Use and understand inductive reasoning and deductive reasoning and understand the differences between them;
- Use geometric and visual reasoning;
- Use multiple representations (e.g., analytic, numerical and geometric) to solve problems. (This is extremely important – the concept that there is not just one correct way to solve a problem);
- Use a variety of strategies to revise solution processes;
- Understand the uses of both proof and counterexample in problem solutions, know how to use and generate counterexamples, are able to write simple proofs;
- Are familiar with the process of creating and understanding mathematical models from word problems, geometric problems and applications and are able to interpret solutions in the context of these source problems;
- Translate simple statements into operations; and
- Understand the role of written symbols in representing mathematical ideas and the precise use of special symbols of mathematics.

**B. Successful students know how to estimate; they**

- Are able to convert between decimal approximations and fractions;
- Know when to use an estimation or approximation in place of an exact answer;
- Recognize the accuracy of an estimation; and
- Know how to make and use estimations.

**C. Successful students understand the appropriate use as well as the limitation of calculators; they**

- Recognize when the results produced are unreasonable or represent misinformation and
- Use calculators for systematic trial-and-error problem solving.

**D. Successful students are able to generalize and to go from specific to abstract and back again. They have a basic understanding of mathematical modeling.**

**E. Successful students demonstrate active participation in the process of learning mathematics; they**

- Are willing to experiment with problems that have multiple solutions;
- Demonstrate an understanding of the importance of the mathematical ideas behind the steps of a solution, as well as the solution;
- Show an understanding of how to modify patterns and solutions strategies to obtain different results; and
- Recognize when a proposed solution does not work, analyze why and use the analysis to seek a valid solution

**F. Successful students recognize the broad range of applications of mathematical reasoning. They have an appreciation of the relevance of mathematical models and know that mathematical applications are used in other fields.**

## II. Computation

### A. Successful students know basic mathematical operations; they

- Understand, perform and apply arithmetic operations with real numbers, and percents and proportions with and without a calculator;
- Understand percent, proportions, and ratios;
- Understand exponents expressed as integer and rational numbers;
- Understand integer exponents expressed as integer and rational numbers;
- Interpret and write scientific notation and understand order of magnitude;
- Understand the basic structure of the real number line;
- Understand absolute value;
- Use the correct order of arithmetic operations, particularly demonstrating facility with the Distributive Law; and
- Know terminology for, and relationships among, natural, integer, rational, irrational and real numbers.

### B. Successful students know and carefully record symbolic manipulations. They understand the uses of mathematical symbols as well as the limitations on their appropriate uses (e.g. equality and inequality signs, greater than and less than signs, parentheses, superscripts and subscripts), write standard notations and convert calculator notations to standard mathematical notation.

## III. Algebra

### A. Successful students know and apply basic algebraic concepts; they

- Understand the concept of a variable;
- Correctly perform addition, subtraction, multiplication and division operations that include variables, with emphasis on grouping like terms;
- Perform appropriate basic operations on sets (e.g., elements of subsets, union, intersection, and complements);
- Use the distributive property to multiply a monomial by a polynomial and to multiply two polynomials;
- Understand exponents, roots and their properties in expressions involving variables;
- Simplify and perform basic operations on rational expressions, including finding common denominators: (add, subtract, multiply, divide);
- *\*Understand the relationship between and properties of common and natural exponentials and logarithms and*
- *\*Factor polynomials: common terms, difference of two squares, and trinomials.*

### B. Successful students are able to work with mathematical notation to solve problems and communicate solutions; they

- Translate simple statements into equations; and
- Understand the role of written symbols in representing mathematical ideas and the precise use of special symbols of mathematics as used in algebra.

### C. Successful students use various appropriate techniques to solve and apply basic equations and inequalities, using algebraic, numeric and graphic methods—*both with and without technology*—and are able to

- Derive, solve, and interpret first degree equations and quadratic equations in one variable;
- Derive, solve, and interpret first degree inequalities in one variable;
- Derive, solve, and interpret systems of linear equations in two variables; and
- *\*Solve quadratic equations using factoring, completing the square and the quadratic formula.*

D. Successful students distinguish between and among expressions, formulas, equations and functions; they

- Understand the difference between simplifying, solving, substituting or evaluating equations and expressions;
- Understand the concept of a function, including domain and range, and notation; and
- Understand that functions can be expressed as verbal statements, numbers, formulas, graphs, and tables.

E. Successful students understand the relationship between equations and graphs; they

- Understand basic forms of the equation of a straight line and how to graph the line without the aid of a calculator;
- Understand the basic shapes of the graphs of quadratic functions and know how to find the vertex of a parabola; and
- Know and understand the basic shape of the graphs of exponential and logarithmic functions.

#### IV. Geometry

A. Successful students understand and use both basic plane and solid geometry; they

- Understand the concepts of area, perimeter, volume and surface area;
- Know how to calculate area, perimeter, volume and surface area;
- Use similar figures to understand the effects of scale on the figure and to find unknown angle measurements and lengths of sides;
- Understand transformations and transformational geometry: rotation, translation, reflection, and dilation; and
- Understand and apply the Pythagorean Theorem.

B. Successful students know linear analytic (i.e. coordinate) geometry; they

- Can find the equation of a line given various properties;
- Understand slope as a rate of change;
- *\*Understand the distance formula and how to find the midpoint of a segment; and*
- *\*Understand the properties of slopes of parallel and perpendicular lines.*

#### V. Data Analysis and Statistics

Successful students apply concepts of statistics and data analysis in the social and natural sciences. They know how to represent data in a variety of ways (e.g., scatter plot, line graph and two-way table) and select the most appropriate. They

- Use various methods of data representation: tables, graphs, numbers;
- Interpret data from various methods of representation;
- Understand mean, median, mode as measures of central tendency and their proper applications;
- Understand the definition of probability and why probability events are expressed as between zero and one inclusive; and
- Understand data dispersion.